IQM3004

LEVEL 3 · CONTROLLED STAFF DOCUMENT

INTO QUALIFICATIONS

Mathematics

Component 2: Paper 2 written examination

STAFF ONLY · Mark scheme

Duration
3 hours

Total marks
150

Assessment window
Sample assessment

Assessment set
Set S of 2026

Audience
Staff only

Release timing
exam release

Exam date
_____

Assessment set ID
2026-S

STAFF ONLY · Use only this assessment-specific scheme and approved standardisation decisions.	
Assessment window Sample assessment	**Exam date** _____
Paper P2	**Assessment set** Sample 2026 · ID 2026-S

General marking principles

- Record every awarded mark in the corresponding OCR-readable annotation on the candidate response document.
- Credit equivalent valid answers only where the approved item-specific scheme permits them.
- Do not infer grade boundaries from raw marks; award boundaries are held in programme specifications.

Each M1 is one method mark, A1 one accuracy mark dependent on the associated method, and B1 one independent mark. Accept equivalent valid methods and exact forms. Correct answers can imply method. Use 3 significant figures for non-exact numerical answers unless another accuracy is specified. No follow-through unless stated. AO labels support assessment mapping.

Item ID	Question / requirement	Solution and allocation	Alternatives / notes	Max
A1(a)	Expand $(1+2x)^5$ through the term in x^3.	B1 (AO1): $1+10x$. M1 (AO1): Use binomial coefficients 10 for x^2 and x^3. A1 (AO1): $1+10x+40x^2+80x^3$.	General principles apply.	3
A1(b)	Find the coefficient of x^4 in $(2-x)^6$.	M1 (AO1): $\binom{6}{4}2^2(-1)^4$. A1 (AO1): 60.	General principles apply.	2
A2(a)	Expand $(1-2x)^{-1/2}$ through the term in x^2.	B1 (AO1): Constant term 1. M1 (AO1): Use $n=-1/2$, so terms $(-1/2)(-2x)$ and $[(-1/2)(-3/2)/2](-2x)^2$. A1 (AO1): $1+x+3x^2/2$.	General principles apply.	3
A2(b)	Use your expansion of $(1-2x)^{-1/2}$ through x^2 to approximate $1/\sqrt{0.96}$.	M1 (AO3): Substitute $x=0.02$. A1 (AO1): 1.02.	General principles apply.	2

A2(c)	State the strict inequality in x required by the standard binomial expansion condition for $(1-2x)^{-1/2}$.	B1 (AO1): $\lvert x\rvert<1/2$.	General principles apply.	1
A3(a)	An arithmetic progression has third term 11 and eighth term 31. Find the first term and common difference.	M1 (AO1): $a+2d=11$, $a+7d=31$. A1 (AO1): $d=4$. A1 (AO1): $a=3$.	General principles apply.	3
A3(b)	Find the least positive integer n for which $\sum_{k=1}^{n}(4k-1)>1000$.	M1 (AO1): Sum $n[2(3)+(n-1)4]/2$. A1 (AO1): $2n^2+n>1000$. M1 (AO2): Solve the boundary or compare consecutive integer values; positive root $(-1+\sqrt{8001})/4$. A1 (AO2): $n=23$, since $S_{22}=990$ and $S_{23}=1081$.	General principles apply.	4
A4(a)	A sequence satisfies $u_1=100$ and $u_{n+1}=0.8u_n+10$. Find u_2 and u_3.	B1 (AO1): $u_2=90$. B1 (AO1): $u_3=82$.	General principles apply.	2
A4(b)	For $u_1=100$, $u_{n+1}=0.8u_n+10$, define $v_n=u_n-50$. Show that v_n is geometric, find a formula for u_n, and find its limit as n increases.	M1 (AO2): $v_{n+1}=0.8u_n+10-50$. A1 (AO2): $v_{n+1}=0.8(u_n-50)=0.8v_n$, with $v_1=50$. A1 (AO1): $u_n=50+50(0.8)^{n-1}$. B1 (AO2): $(0.8)^{n-1}\to 0$ because $\lvert 0.8\rvert<1$. A1 (AO1): Limit 50.	General principles apply.	5
A5(a)	Solve $3e^{2x}=15$, giving an exact value of x.	M1 (AO1): $e^{2x}=5$. M1 (AO1): $2x=\ln 5$. A1 (AO1): $x=(\ln 5)/2$.	General principles apply.	3

A5(b)	Sketch $y=\ln(x-2)$, labelling its vertical asymptote and x-intercept.	B1 (AO1): Increasing logarithmic curve with vertical asymptote $x=2$. B1 (AO1): Crosses at $(3,0)$; graph exists only for $x>2$.	General principles apply.	2
A5(c)	Solve $\log_2(x-1)+\log_2(x+1)=3$, checking the domain.	M1 (AO1): $\log_2(x^2-1)=3$, hence $x^2-1=8$. A1 (AO1): Algebraic candidates $x=3,-3$. B1 (AO2): Only $x=3$ is valid because both arguments must be positive, requiring $x>1$.	General principles apply.	3
A6(a)	Prove that $(sec^2\theta-1)/\tan\theta=\tan\theta$ wherever the left-hand expression is defined.	M1 (AO2): Use $sec^2\theta-1=\tan^2\theta$. A1 (AO2): Divide by nonzero $\tan\theta$ to obtain $\tan\theta$. B1 (AO2): Domain requires $\cos\theta\neq0$ and $\tan\theta\neq0$; the manipulation respects these restrictions.	General principles apply.	3
A6(b)	Solve $2\sin^2\theta-3\sin\theta+1=0$ for $0\le\theta<2\pi$.	M1 (AO1): $(2\sin\theta-1)(\sin\theta-1)=0$. A1 (AO1): $\sin\theta=1/2$ or 1. B1 (AO1): $\theta=\pi/6,5\pi/6$. B1 (AO1): $\theta=\pi/2$; exactly these three solutions.	General principles apply.	4
A6(c)	Write $3\sin\theta+4\cos\theta$ as $R\sin(\theta+\alpha)$, where $R>0$ and $0<\alpha<\pi/2$. State its maximum value.	M1 (AO1): Match $R\cos\alpha=3$, $R\sin\alpha=4$. A1 (AO1): $R=5$. A1 (AO1): $\alpha=\arctan(4/3)\approx0.927$ radians. B1 (AO2): Maximum is 5 because sine has maximum 1.	General principles apply.	4
A7(a)	Let $f(x)=2x+1$ on $\mathbb{R}$ and $g(x)=x^2$ on $\mathbb{R}$. Find $f(g(x))$ and its range.	M1 (AO1): Substitute $g(x)$ into f. A1 (AO1): $f(g(x))=2x^2+1$. B1 (AO2): Range $[1,\infty)$ since $x^2\ge0$, with minimum 1 attained at x = 0.	General principles apply.	3

A7(b)	For $h(x)=x^2+3$ restricted to $x\ge 0$, find the inverse function, stating its domain and range.	M1 (AO1): Swap x, y and solve $x=y^2+3$ using the nonnegative branch. A1 (AO1): $h^{-1}(x)=\sqrt{x-3}$, domain $x\ge 3$. B1 (AO2): Range $y\ge 0$, consistent with the original domain.	General principles apply.	3
A8(a)	Sketch $y=2\lvert x-3\rvert-4$, labelling the vertex and all axis intercepts.	B1 (AO1): Upward V with vertex $(3,-4)$. B1 (AO1): x-intercepts $(1,0)$ and $(5,0)$. B1 (AO1): y-intercept $(0,2)$ and slopes $-2,+2$.	General principles apply.	3
A8(b)	Solve $\lvert 2x-1\rvert<5$.	M1 (AO1): $-5<2x-1<5$. A1 (AO1): $-2<x<3$.	General principles apply.	2
A9(a)	For small x measured in radians, use small-angle approximations to simplify $\sin(3x)/\tan(2x)$. State the limiting value as $x\to 0$.	M1 (AO1): Replace numerator by $3x$ and denominator by $2x$. A1 (AO2): Approximation and limit $3/2$.	General principles apply.	2
A9(b)	Sketch $y=\sec x$ for $-\pi<x<\pi$, showing its asymptotes and intercept with the y-axis.	B1 (AO1): Vertical asymptotes $x=\pm\pi/2$. B1 (AO1): Central positive branch with minimum $(0,1)$. B1 (AO2): Negative outer branches below -1, diverging to $-\infty$ at adjacent asymptotes and tending to -1 at the excluded endpoints.	General principles apply.	3
B1(a)	Differentiate $y=x^2e^{3x}$ and factorise your answer. State the two differentiation rules used.	M1 (AO1): Product rule gives $2xe^{3x}+x^2(d/dx)(e^{3x})$. A1 (AO1): Chain rule gives $(d/dx)e^{3x}=3e^{3x}$. A1 (AO1): $dy/dx=xe^{3x}(2+3x)$. B1 (AO2): Names product and chain rules.	General principles apply.	4

B1(b)	For $y=\ln x/x$ with $x>0$, find dy/dx and determine the interval on which y is increasing.	M1 (AO1): Quotient rule: $[x(1/x)-\ln x]/x^2$. A1 (AO1): $dy/dx=(1-\ln x)/x^2$. M1 (AO2): Since denominator is positive, require $1-\ln x>0$. A1 (AO2): $0<x<e$.	General principles apply.	4
B1(c)	The curve $x^2+xy+y^2=7$ passes through $(1,2)$. Find dy/dx by implicit differentiation and evaluate it at $(1,2)$.	M1 (AO1): $2x+y+x(dy/dx)+2y(dy/dx)=0$. A1 (AO1): $dy/dx=-(2x+y)/(x+2y)$. A1 (AO1): At $(1,2)$, gradient $-4/5$.	General principles apply.	3
B1(d)	Find the stationary points of $y=x^2e^{3x}$ and determine their nature.	M1 (AO1): $xe^{3x}(2+3x)=0$, so $x=0,-2/3$. A1 (AO1): Points $(0,0)$ and $(-2/3,4/(9e^2))$. B1 (AO2): Derivative is positive for $x<-2/3$, negative for $-2/3<x<0$, positive for $x>0$ (or valid second derivative test). A1 (AO2): Maximum at $(-2/3,4/(9e^2))$ and minimum at $(0,0)$.	General principles apply.	4
B2(a)	Use substitution $u=x^2+1$ to evaluate $\int_0^1 2x\sqrt{x^2+1}\ dx$.	M1 (AO1): $du=2x\ dx$. B1 (AO1): New limits $u=1,2$. M1 (AO1): Integrate $u^{1/2}$ to $(2/3)u^{3/2}$. A1 (AO1): $(2/3)(2\sqrt{2}-1)$.	General principles apply.	4
B2(b)	Find $\int \cos(3x)\ dx$.	M1 (AO1): Reverse the chain rule with factor $1/3$. A1 (AO1): $\frac{1}{3}\sin(3x)+C$.	General principles apply.	2
B2(c)	Find $\int xe^x\ dx$ by integration by parts.	M1 (AO1): Choose $u=x$, $dv=e^x dx$, so $du=dx$, $v=e^x$. M1 (AO1): $xe^x-\int e^x dx$. A1 (AO1): $(x-1)e^x+C$.	General principles apply.	3

B2(d)	Find the exact area bounded by $y=e^x$, $y=1$, $x=0$ and $x=\ln 2$.	M1 (AO3): Form area integral $\int_0^{\ln 2}(e^x-1)\,dx$. A1 (AO1): Antiderivative e^x-x. M1 (AO1): Substitute limits: $(2-\ln 2)-(1-0)$. A1 (AO1): $1-\ln 2$ square units.	General principles apply.	4
B2(e)	Find $\int 3/(3x+2)\,dx$, stating the intervals on which your expression is valid.	B1 (AO1): $\ln\lvert 3x+2\rvert+C$. B1 (AO2): On either interval $x<-2/3$ or $x>-2/3$ separately; the integrand is undefined at $-2/3$.	General principles apply.	2
B3(a)	Points A,B,C have position vectors $(1,2,-1)$, $(5,0,3)$ and $(3,5,1)$. Find $\overrightarrow{AB}$, its magnitude, and the position vector of the midpoint of AB.	M1 (AO1): Subtract position vector of A from B. A1 (AO1): $\overrightarrow{AB}=(4,-2,4)$. M1 (AO1): Magnitude $\sqrt{4^2+(-2)^2+4^2}$. A1 (AO1): Magnitude 6. B1 (AO1): Midpoint $(3,1,1)$.	General principles apply.	5
B3(b)	$A=(1,2,-1)$ and $B=(5,0,3)$. Point P divides AB internally in the ratio $AP:PB=1:3$. Find P and show that P lies in the plane $z=0$.	M1 (AO3): Use $\overrightarrow{OP}=\overrightarrow{OA}+\frac{1}{4}\overrightarrow{AB}$. A1 (AO1): $P=(2,3/2,0)$. B1 (AO2): Internal ratio uses one quarter of AB, not one third. A1 (AO2): Its z-coordinate is zero, hence it lies in the stated plane.	General principles apply.	4
B3(c)	Points $A=(1,2,-1)$, $B=(5,0,3)$ and $C=(3,5,1)$ are consecutive vertices of parallelogram $ABCD$. Find D and verify that the diagonals have the same midpoint.	M1 (AO3): $D=A+C-B$. A1 (AO1): $D=(-1,7,-3)$. B1 (AO1): Midpoint AC is $(2,7/2,0)$. A1 (AO2): Midpoint BD is also $(2,7/2,0)$, verifying the required property.	General principles apply.	4

B3(d)	A displacement in a horizontal plane is $3i+4j$ km, where i points east and j north. Find its magnitude and bearing to the nearest degree.	B1 (AO1): Magnitude 5 km. B1 (AO3): Bearing 037°, since the angle east of north is $\arctan(3/4)\approx 36.87^\circ$.	General principles apply.	2
B4(a)	A vehicle starts from rest. Its velocity increases uniformly to 12 m/s in 6 s, stays at 12 m/s for 10 s, then decreases uniformly to rest in 4 s. Sketch the velocity-time graph and find the total distance travelled.	B1 (AO1): Labelled axes t in seconds and v in m/s; straight segment from (0,0) to (6,12). B1 (AO1): Horizontal segment to (16,12), then straight segment to (20,0). M1 (AO3): Distance is area under the nonnegative velocity graph. M1 (AO1): $\frac{1}{2}(6)(12)+(10)(12)+\frac{1}{2}(4)(12)$. A1 (AO3): 180 m.	General principles apply.	5
B4(b)	A ball is projected vertically upwards at 19.6 m/s from ground level. Neglect air resistance and use $g=9.8$ m/s². Find the time to maximum height, the maximum height, and the total time before it returns to ground level.	M1 (AO3): At the top $v=0$, so $0=19.6-9.8t$. A1 (AO1): Time to top 2 s. M1 (AO1): Use $v^2=u^2+2as$ or displacement formula. A1 (AO3): Maximum height 19.6 m. A1 (AO3): Total time 4 s, either by symmetry or positive root of $19.6t-4.9t^2=0$.	General principles apply.	5
B4(c)	A particle is acted on by forces $F_1=(3,4)$ N and $F_2=(-1,2)$ N in the horizontal plane. Find their resultant. Find the third force required for equilibrium, sketch the three force vectors acting at the particle, and explain what equilibrium implies about its motion.	M1 (AO1): Add corresponding components. A1 (AO1): Resultant $(2,6)$ N. A1 (AO3): Third force $(-2,-6)$ N. B1 (AO2): Labelled force diagram with the three vectors in consistent directions, concurrent at the particle. B1 (AO2): Zero resultant means constant velocity, including rest; equilibrium does not require the particle already to be stationary.	General principles apply.	5

B5(a)	For $f(x)=x^3-x-2$, show that a root lies between 1 and 2, explaining the mathematical condition that makes this argument valid.	B1 (AO1): $f(1)=-2$. B1 (AO1): $f(2)=4$. B1 (AO2): Polynomial is continuous; opposite signs imply at least one intervening root.	General principles apply.	3
B5(b)	Use $x_{n+1}=(x_n+2)^{1/3}$ with $x_0=1.5$ to calculate x_1 and x_2, retaining at least six decimal places during working and reporting four decimal places.	M1 (AO1): Substitute 1.5 into recurrence. A1 (AO1): $x_1=1.5183$ (unrounded 1.518294486). A1 (AO1): $x_2=1.5209$ (unrounded about 1.520935).	General principles apply.	3
B5(c)	Write the Newton-Raphson iteration for $f(x)=x^3-x-2$ and calculate one step from $x_0=1.5$.	B1 (AO1): $f'(x)=3x^2-1$. M1 (AO1): $x_{n+1}=x_n-(x_n^3-x_n-2)/(3x_n^2-1)$. A1 (AO1): $x_1=1.521739130\ldots$, or 1.52 to three significant figures.	General principles apply.	3
B5(d)	Given $f(1.5213)=-0.000473731403$ and $f(1.5214)=0.000120620344$ for $f(x)=x^3-x-2$, justify the root to three decimal places.	M1 (AO2): Continuity and opposite signs enclose a root in $(1.5213, 1.5214)$. B1 (AO2): Every value in this interval rounds to 1.521 to three decimal places. A1 (AO2): The enclosed root is therefore 1.521 to three decimal places.	General principles apply.	3
B5(e)	A student proposes using Newton-Raphson for $f(x)=x^3-x-2$ with $x_0=1/\sqrt{3}$. Explain why the first step cannot be calculated, and suggest a suitable alternative starting value supported by information in this question.	M1 (AO2): $f'(1/\sqrt{3})=0$. A1 (AO2): The iteration would divide by zero; the tangent is horizontal. B1 (AO3): Use 1.5, which lies in the bracket (1,2), has nonzero derivative and yielded a valid step in part (c).	General principles apply.	3

B6(a)	A machine costs £24000. Its resale value is modelled by $V_n = 24000(0.85)^n$ pounds after n complete years. Find its value after 3 years and interpret the factor 0.85.	M1 (AO1): Substitute $n = 3$. A1 (AO1): £14739. B1 (AO3): The model assumes a 15% loss in value each year, applied to the previous value.	General principles apply.	3
B6(b)	For $V_n = 24000(0.85)^n$, find the first complete year at which the model predicts a value below £8000. Show your logarithmic working.	M1 (AO3): Solve $(0.85)^n < 1/3$. M1 (AO1): Take logs: $n \ln 0.85 < \ln(1/3)$. A1 (AO2): Reverse inequality because $\ln 0.85 < 0$: $n > \ln(1/3)/\ln 0.85 \approx 6.76$. A1 (AO3): First complete year is 7.	General principles apply.	4
B6(c)	The machine saves £5000 in year 1, £4500 in year 2 and continues with annual savings multiplied by 0.9. Find the total modelled savings in the first 5 years and the infinite total. Explain why the infinite total may not be a useful prediction.	M1 (AO1): Finite geometric sum $5000(1 - 0.9^5)/(1 - 0.9)$. A1 (AO1): £20475.50. A1 (AO1): Infinite sum $5000/(1 - 0.9) =$ £50000. B1 (AO3): For example, machine life is finite or savings will not follow the same ratio indefinitely; connect the limitation to the infinite-horizon assumption.	General principles apply.	4
B6(d)	The owner plans to sell after 5 years. Using resale model $V_n = 24000(0.85)^n$ and annual savings $5000(0.9)^{n-1}$, calculate total savings plus resale value minus the £24000 purchase cost. Ignore interest and maintenance. State one additional cost or uncertainty needed for a real investment decision.	M1 (AO3): Combine five-year savings and fifth-year resale, then subtract purchase price. A1 (AO1): Resale £10648.9275; savings £20475.50. A1 (AO3): Net £7124.4275, or £7124.43 to nearest penny; also accept £7120 to 3 significant figures. B1 (AO3): Relevant omitted factor, e.g. maintenance, finance costs, downtime or uncertainty in resale/savings; a specific factor required.	General principles apply.	4